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QUADRATIC INEQUALITIES

Quadratic Inequalities: GRAPHIC METHOD

We start from a quadratic inequality: $2x^2 - 7x - 15 \geq 0$

This inequality is written in normal form.

Step 1: The first thing to do is to find the solution of the equation $2x^2 - 7x - 15 = 0$. We can proceed with the A method:

We substitute the letters a, b, c with the numbers of the equation:

a = it represents the number that multiplies $x^2 = 2$

b = it represents the number that multiplies $x = -7$

c = it is the free term = -15

At this point we determine the A

Step 2: The formula of A is:

$$A = b^2 - 4ac$$

$$= 49 - 4 \cdot 2 \cdot (-15)$$

$$= 49 + 120$$

$$= 169$$

169 is a positive number, so we expect real and positive solutions

$$x_{1,2} = \frac{-b \pm \sqrt{A}}{2a}$$

$$= \frac{-7 \pm \sqrt{169}}{4}$$

$$x_1 = \frac{7+13}{4} = 5$$

$$x_2 = \frac{7-13}{4} = -\frac{3}{2}$$

It would be more precise to write $x_1 = -\frac{3}{2}$ and $x_2 = 5$ because usually (especially when we use the

GRAPHIC METHOD) x_1 IS SMALLER THAN x_2 , SO:

$$x_1 = -\frac{3}{2}; x_2 = 5$$

THESE ARE THE SOLUTIONS OF THE QUADRATIC EQUATION!
AT THIS POINT WE PROCEED TO SOLVE THE INEQUALITY WITH
THE GRAPHIC METHOD.

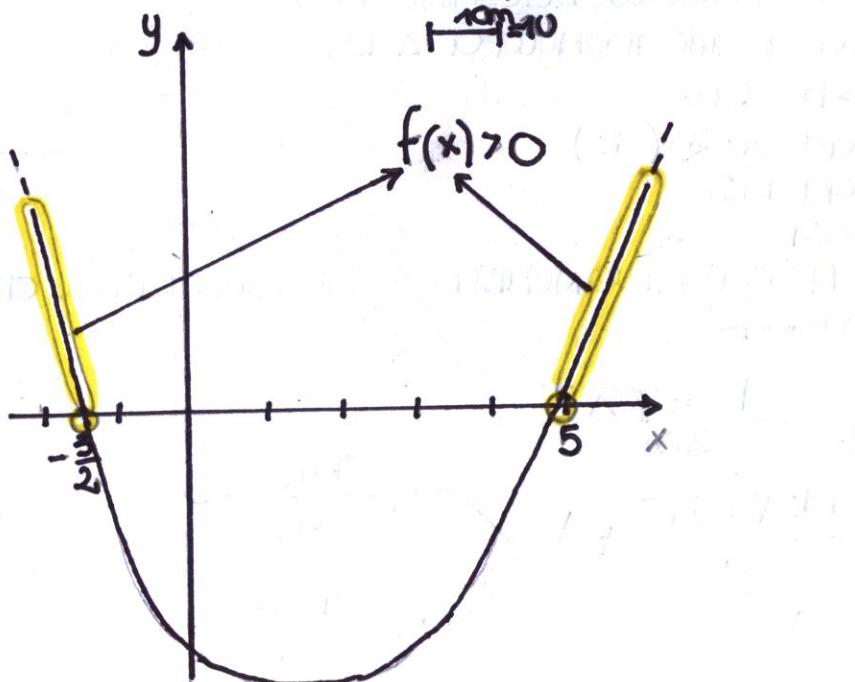
THE GRAPHIC METHOD CONSIST IN DRAWING A STYLIZED GRAPH
OF THE EQUATION PARABOLA: $y = 2x^2 - 7x - 15$

IT IS STYLIZED BECAUSE THE ONLY TWO ELEMENTS THAT MUST
REALLY BE PRECISE IN THE DRAWING ARE:

1- THE CONCAVITY OF THE PARABOLA

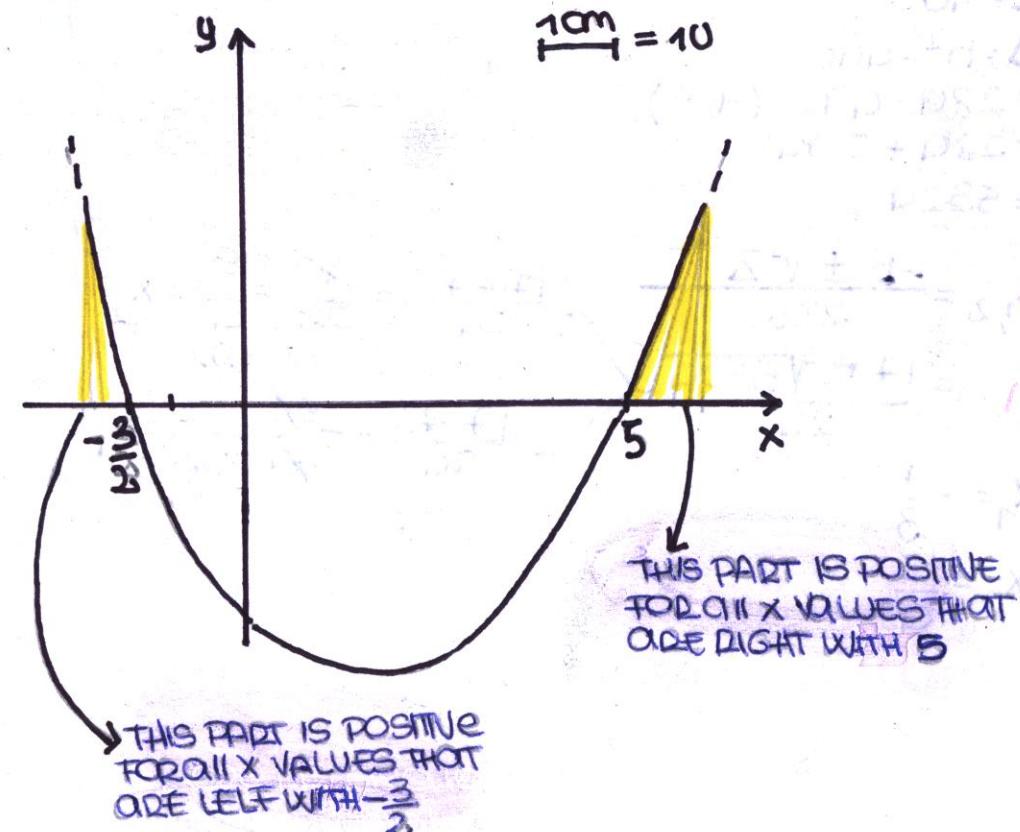
2- THE INTERSECTIONS WITH THE AXIS X.

THESE INTERSECTIONS ARE OBTAINED IN CORRESPONDENCE
WITH THE SOLUTIONS OF THE EQUATION SO THE PARABOLA
WILL PASS FOR THESE TWO POINTS $(-\frac{3}{2}, 0), (5, 0)$



THE CONCAVITY IS DETERMINED BY THE SIGN OF THE
MAXIMUM DEGREE COEFFICIENT, SO, THE NUMBER THAT
MULTIPLIES x^2 (IN THIS CASE IT IS 2). IF THIS NUMBER IS
POSITIVE THE PARABOLA WILL HAVE UPWARD CONCAVITY,
IF IT IS NEGATIVE THE PARABOLA WILL HAVE DOWNWARDS
CONCAVITY. THE INEQUALITY ASKS TO DETERMINE THE VALUES
OF X FOR WHICH THIS POLYNOMIAL IS ≥ 0 WE HAVE TO
LOOK FOR WHICH VALUES OF X THE PARABOLA IS POSITIVE.
OR WHEN IT IS IN THE POSITIVE HALF PLANE OF THE
CARTESIAN PLANE

FOR WHICH VALUES OF X THE QUADRATIC INEQUALITY
 $2x^2 - 7x - 15 \geq 0$?



THE CONCLUSION IS:

$$x \leq -\frac{3}{2} \vee x \geq 5$$

Examples of quadratic inequalities with graphic method

$$12x^2 - 17x - 105 < 0$$

$$12x^2 - 17x - 105 = 0$$

$$a = 12$$

$$b = -17$$

$$c = -105$$

$$\Delta = b^2 - 4ac$$

$$= 289 - 4 \cdot 12 \cdot (-105)$$

$$= 289 + 5040$$

$$= 5329$$

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= \frac{17 \pm \sqrt{5329}}{24}$$

$$\frac{17+73}{24} = \frac{90}{24} = \frac{15}{4} = x_2$$

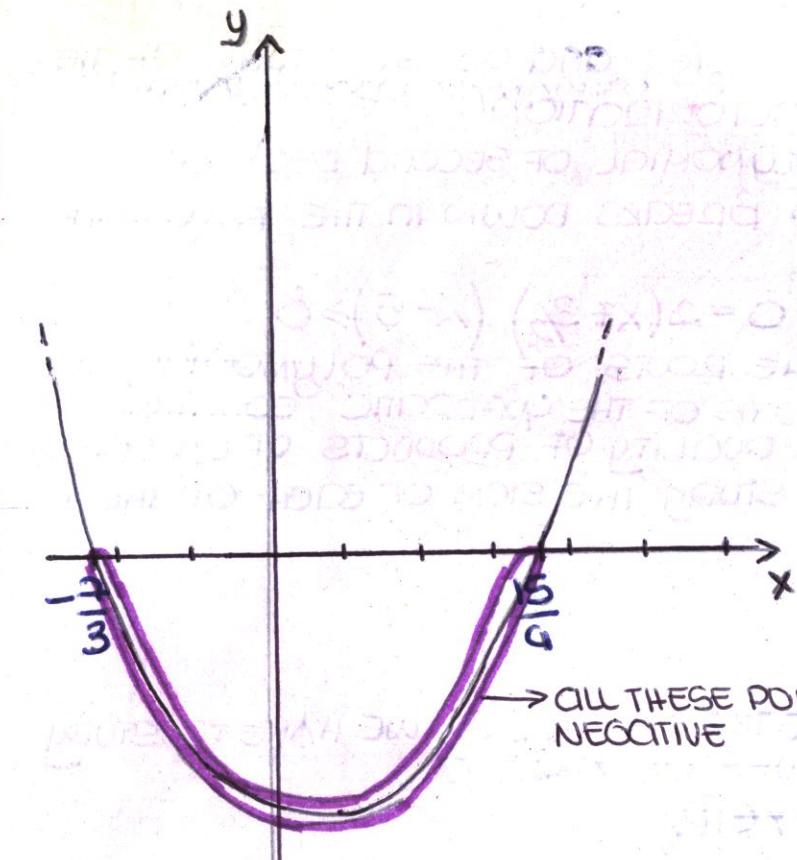
$$\frac{17-73}{24} = \frac{-56}{24} = -\frac{7}{3} = x_1$$

$$x_1 = -\frac{7}{3}$$

$$x_2 = \frac{15}{4}$$

$$y = 12x^2 - 17x - 105$$

graph



$$\text{Solutions: } -\frac{7}{3} < x < \frac{15}{4}$$

Algebraic Method

THE SOLUTION THROUGH THE ALGEBRAIC METHOD IS BASED ON THE BREAK-DOWN OF THE POLYNOMIAL IN THE SECOND DEGREE AND ON THE STUDY OF THE SIGN OF THIS FACTORIZATION

THE GENERIC POLYNOMIAL OF SECOND DEGREE $2x^2 - 7x - 15 \geq 0$ BREAKS DOWN IN THE FOLLOWING WAY:

$$2x^2 - 7x - 15 \geq 0 = 2(x + \frac{3}{2})(x - 5) \geq 0$$

$(-\frac{3}{2}, 5)$ ARE THE ROOTS OF THE POLYNOMIAL, SO THE SOLUTIONS OF THE QUADRATIC EQUATION HAVING THE INEQUALITY OF PRODUCTS OF ALGEBRAIC QUANTITIES WE STUDY THE SIGN OF EACH OF THESE QUANTITIES

$$2(x + \frac{3}{2})(x - 5)$$

I II III

TO MAKE THE PATTERN OF SIGNS WE HAVE TO STUDY FOR WHICH VALUES OF x $2 \geq 0$

$$\text{I } 2 \geq 0 \quad \forall x \in \mathbb{R}$$

$$\text{II } x + \frac{3}{2} \geq 0 \quad x \geq -\frac{3}{2}$$

$$\text{III } x - 5 \geq 0 \quad x \geq 5$$

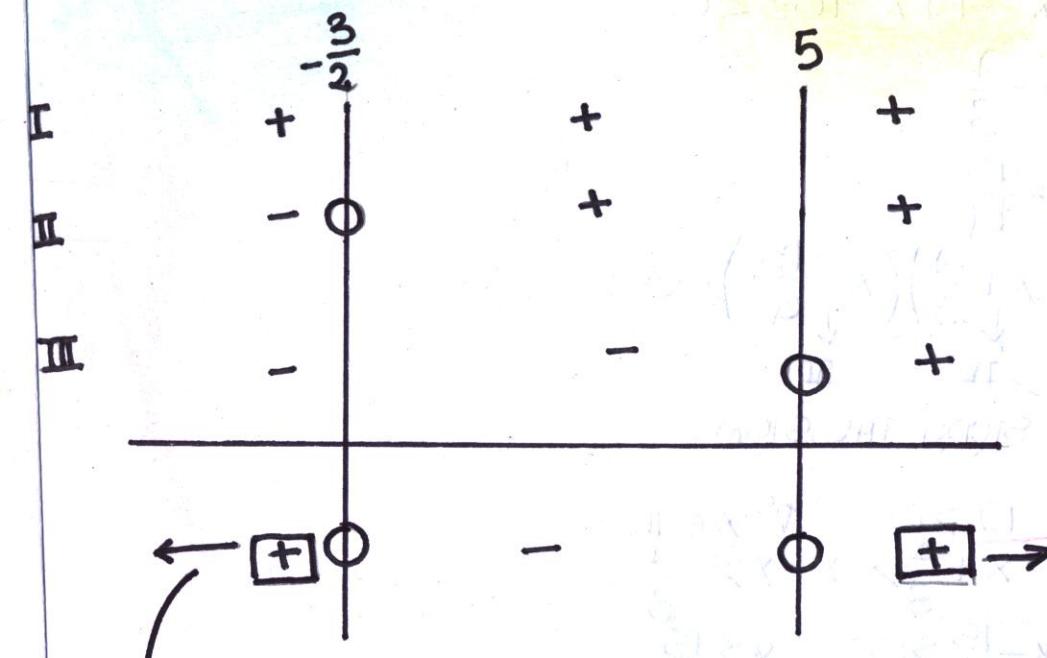
WE DETERMINE THE STUDY OF SIGN (INTERVALS OF POSITIVES VALUES AND INTERVALS OF NEGATIVE VALUES) WE PUT IN ORDER THE QUANTITIES WHO CAME OUT ACCORDING TO A GROWING ORDER.

IF $\Delta > 0$

IN THIS CASE WE HAVE 2 REALS AND SEPARATE SOLUTIONS FOR THE ASSOCIATED EQUATION

THE POLYNOMIAL ASSUMES:

- POSITIVES VALUES IN THE EXTERNAL INTERVALS IDENTIFIED FROM THE 2 SOLUTIONS
- NEGATIVES VALUES IN THE INTERNAL INTERVAL



WE HAVE TO DO THE PRODUCT OF SIGNS

E TRANSLATE:

$$x \leq -\frac{3}{2} \quad \vee \quad x \geq 5$$

Examples of algebraic method

THE EXAMPLE IS THE SAME OF THE QUADRATIC METHOD:

$$12x^2 - 17x - 105 < 0$$

$$x_1 = -\frac{7}{3}$$

$$x_2 = \frac{15}{4}$$

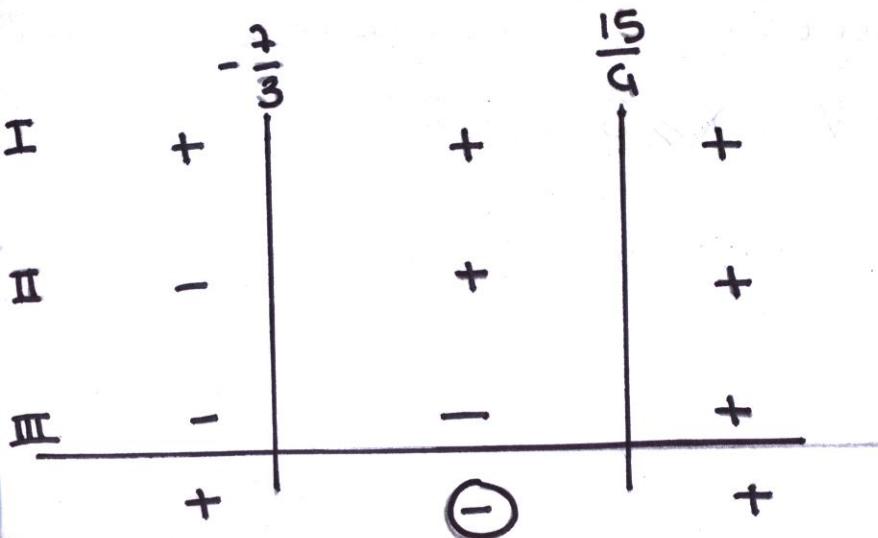
$$12(x + \frac{7}{3})(x - \frac{15}{4}) < 0$$

WE STUDY THE SIGN:

$$\text{I } 12 > 0 \quad \forall x \in \mathbb{R}$$

$$\text{II } x + \frac{7}{3} > 0 \quad x > -\frac{7}{3}$$

$$\text{III } x - \frac{15}{4} > 0 \quad x > \frac{15}{4}$$



$$S = -\frac{7}{3} < x < \frac{15}{4}$$